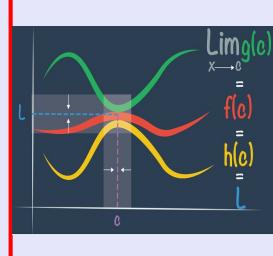


# Math 261

## Spring 2023

### Lecture 35



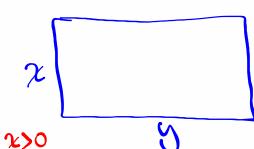
Feb 19-8:47 AM

Find dimensions of a rectangle with area

$1000 \text{ m}^2$  whose perimeter is as small as

Possible.

$$A = 1000$$



$$xy = 1000 \rightarrow y = \frac{1000}{x}$$

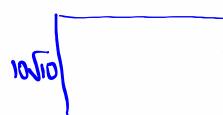
$$P = 2x + 2y \quad \text{Minimum}$$

$$2x + 2\left(\frac{1000}{x}\right)$$

$$f(x) = 2x + \frac{2000}{x}$$

$$f'(x) = 2 - \frac{2000}{x^2}$$

$$f''(x) = \frac{4000}{x^3} > 0 \quad \text{C.U.}$$



$$2 - \frac{2000}{x^2} = 0 \quad \frac{2000}{x^2} = 2 \rightarrow x^2 = 1000$$

$$x = 10\sqrt{10}$$

$$y = \frac{1000}{x} = \frac{1000}{10\sqrt{10}} = \frac{100\sqrt{10}}{\sqrt{10}} \Rightarrow y = 10\sqrt{10}$$

Apr 17-9:52 AM

If  $f(x)$  is differentiable at  $x=a$ , then

$$f(x) \text{ is cont. at } x=a.$$

Recall  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Since  $f(x)$  is differentiable at  $a$ , then

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

If we let  $x=a+h$ , then  $h=x-a$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$f(x) = f(x) - f(a) + f(a)$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [f(x) - f(a) + f(a)]$$

$$= \lim_{x \rightarrow a} [f(x) - f(a)] + \lim_{x \rightarrow a} f(a)$$

$$= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot (x-a) + f(a)$$

$$= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \rightarrow a} (x-a) + f(a)$$

$$= (f'(a)) \cdot (a-a) + f(a)$$

$$\lim_{x \rightarrow a} f(x) = f(a) \Rightarrow f(x) \text{ is continuous at } x=a.$$

Differentiability  $\Rightarrow$  Continuity

Apr 18-8:53 AM

Rolle's Theorem:

- 1)  $f(x)$  is cont. on  $[a, b]$
- 2)  $f(x)$  is diff. on  $(a, b)$
- 3)  $f(a) = f(b)$

Then there is a number  $c$  in  $(a, b)$  such that

$$f'(c) = 0$$

$$\text{ex: } f(x) = x^3 - x^2 - 6x + 2, [0, 3]$$

$f(x)$  is polynomial  $\Rightarrow$  cont.  $\in$  diff. everywhere,

$$f(0) = 0^3 - 0^2 - 6(0) + 2 = 2, f(3) = 3^3 - 3^2 - 6(3) + 2$$

$$= 27 - 9 - 18 + 2$$

$$= 18 - 18 + 2 = 2$$

$$\text{So } f(0) = f(3)$$

by Rolle's Thrm,  $f'(c) = 0$

$c$  is in  $(0, 3)$  Yes

$$f'(x) = 3x^2 - 2x - 6$$

$$P c = \frac{2 + \sqrt{76}}{6} \approx 1.186$$

$$f'(c) = 3c^2 - 2c - 6 = 0$$

$$c = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)} = \frac{2 \pm \sqrt{76}}{6}$$

Apr 18-9:04 AM

Verify all conditions by Rolle's Theorem, then find a number that satisfy the conclusion of Rolle's Thrm for  $f(x) = 5 - 12x + 3x^2$  on  $[1, 3]$ .

$f(x) = 5 - 12x + 3x^2$

↗ Polynomial  
Cont. & diff. everywhere

$$f(1) = 5 - 12(1) + 3(1)^2 = -4 \quad f(3) = 5 - 12(3) + 3(3)^2 = -4$$

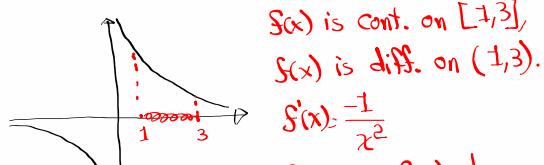
All conditions for Rolle's thrm are met,  
Conclusion  $f'(c) = 0$  for  $c$  in  $(1, 3)$ .

$$f'(x) = -12 + 6x \quad \rightarrow -12 + 6c = 0$$

$$f'(c) = -12 + 6c \quad \boxed{c=2}$$

Apr 18-9:14 AM

## Mean-Value Theorem

1)  $f(x)$  is cont. on  $[a, b]$ 2)  $f(x)$  is diff. on  $(a, b)$ then there is a number  $c$  in  $(a, b)$ such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .Consider  $f(x) = \frac{1}{x}$  on  $[1, 3]$ 

$$f'(c) = \frac{f(b) - f(a)}{b - a} \rightarrow \frac{-1}{c^2} = \frac{\frac{1}{3} - 1}{3 - 1}$$

$$\frac{1}{c^2} = \frac{1/3 - 1}{2} \rightarrow \frac{1}{c^2} = \frac{-2/3}{2} \rightarrow \frac{2}{3}c^2 = 2 \rightarrow c^2 = 3 \rightarrow c = \pm\sqrt{3}$$

$$c = \sqrt{3} \approx 1.732$$

$1.732$  is in  $(1, 3)$

Apr 18-9:20 AM

Verify all conditions for MVT for the function  $f(x) = 2x^2 - 3x + 1$  on  $[0, 2]$ ,

then find all numbers  $c$  that satisfy the conclusion of the MVT.

$$f(x) = 2x^2 - 3x + 1 \quad \begin{array}{l} \text{Polynomial} \\ \text{Cont. \& Diff everywhere} \end{array}$$

$$f(0) = 1 \quad f(2) = 3$$

$$f'(x) = 4x - 3$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$4c - 3 = \frac{3 - 1}{2 - 0} \quad 4c - 3 = \frac{2}{2}$$

$$\begin{array}{l} 4c - 3 = 1 \\ c = 1 \end{array}$$

is in  $(0, 2)$

Apr 18-9:27 AM

Find the area of the largest rectangle that can be inscribed in the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

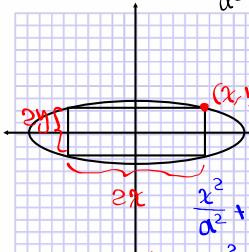
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Centered at  $(0, 0)$  $x$ -Ints  $(\pm a, 0)$  $y$ -Int  $(0, \pm b)$ 

Symmetric

1)  $x$ -axis2)  $y$ -axis

3) Origin



$$\text{Area} = 4xy$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\text{Area} = 4xy$$

$$= 4x \cdot \frac{b}{a} \sqrt{a^2 - x^2}$$

$$f(x) = \frac{4b}{a} x \sqrt{a^2 - x^2}$$

$$f'(x) =$$

$$f''(x) =$$

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right)$$

$$= b^2 \left(\frac{a^2}{a^2} - \frac{x^2}{a^2}\right)$$

$$= \frac{b^2}{a^2} (a^2 - x^2)$$

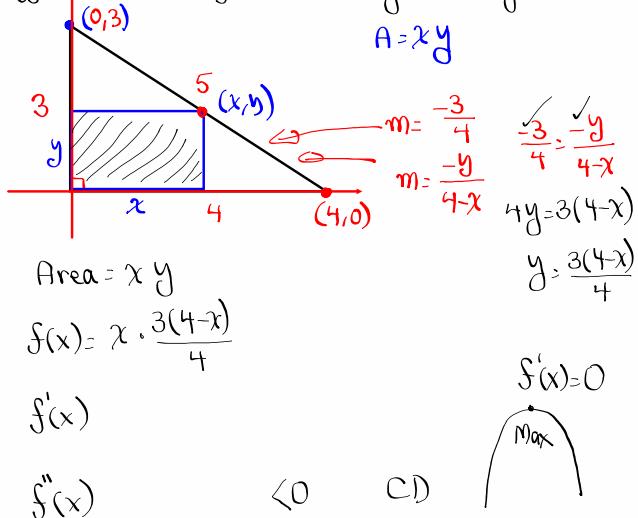
$$y \cdot \frac{b}{a} \sqrt{a^2 - x^2}$$

$$f'(x) = 0$$

$$f''(x) < 0 \quad \text{C.D.} \quad \nearrow \text{Max}$$

Apr 18-9:34 AM

Find the area of the **largest rectangle** that can be inscribed in a right triangle with legs of 3cm and 4cm if two sides of the rectangle lie along the legs.



Apr 18-9:46 AM